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Samuelson's Theory of Business Cycle (Multiplier-Accelerator Model)

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Abstract

We arrive at the process of income generation through the working together of multiplier and accelerator principles. However, Paul Samuelson went further and stated that through the values of multiplier and accelerator, it is possible to explain the path of fluctuations in the pattern of income level and on that basis, he defined five cases of business cycle. Paul Samuelson stated that if we know autonomous consumption, autonomous investment, consumption for the current period and preceding period, we can determine the growth path of income and output for any period with the given values of marginal propensity to consume and capital-output ratio or accelerator by substituting these values in the equation of income (Y). In this paper, the author has examined the theory of business cycle by putting different values of marginal propensity to consume and accelerator in equation and arrived income is plotted on the graph to see if business cycle is formed in the same way as Paul Samuelson has stated. The figure of business cycle that arrived from the combination of marginal propensity to consume and accelerator is somewhat different from that of Samuelson's business cycle.

Keywords: multiplier, accelerator, business cycle, income, autonomous investment.

JEL: E32

Introduction

John Maynard Keynes in his book "The General Theory of Employment, Interest, and Money (1936)" argued that business cycles (Keynes used trade cycle for the term business cycles) are the result of investment changes generated by cyclical changes in the marginal efficiency of capital, a process often become more complicated and often reinforced by associated changes in other short period variables of the economic system (http://eslm.kkhsou.ac.in/ESLM). In his explanation of the business cycle, Keynes used multiplier to explain cyclical fluctuation and changes in income and output. For explanation of the business cycle, Keynes neither used accelerator nor cumulative nature of fluctuations in the economic activity. It was P.A. Samuelson who in his seminal paper "Interaction between the Multiplier Analysis and the Principle of

Acceleration" (1939) combined the newly arrived Keynesian multiplier analysis with the older principle of acceleration (Allen, RGD; 1959). Through interaction of multiplier and acceleration principles, Samuelson developed modern business cycle in 1939. The important point in the Samuelson model of business cycle is enumerated below:

1. An increase in autonomous investment leads to an amplified rise in income, which is determined by the value of the multiplier (k).
2. The instability in the economy is caused by fluctuations, which per se is determined by induced investment, which depends on private investment.
3. Increase in income increases aggregate demand for goods and services, which depends on multiplier.
4. Increase in aggregated demand for goods and services increases income demand for capital goods, which is required to produce consumption goods. Demand for capital goods is induced investment which depends on accelerator (i.e., capital-output ratio).
5. Without any external shocks accelerator and multiplier interactions can result in business cycles whose pattern is shaped by the magnitude of capital-output ratio and marginal propensity to consume.

Assumptions of the Samuelson's Business Cycle Theory

Samuelson's theory of business cycle is based on the following assumptions:

1. There is no excess capacity of production; i.e., production capacity is limited;
2. Combination of multiplier and accelerator causes economic fluctuation in the economy;
3. It is income of the preceding year that determines current period consumption spending;
4. Change in income with a lag of one period determine current period investment;
5. There is closed economy, i.e. no foreign trade;
6. There would be gap of one year between the increase in consumption and increase in demand for investment;
7. The exogenous spending, i.e., government spending is constant and no attempt is made to use government spending as an instrument for controlling the time path of income;
8. Marginal propensity to consume (c) is greater than zero but less than one. This may be written as $0 < c < 1$ and accelerator is greater than 1; and
9. This model neither considers the effects of public spending on the level of income and output not determinants of the government spending.

Explanation of Samuelson's Business Cycle Theory

According to Samuelson, multiplier per se is inefficient to explain cumulative and the cyclical nature of the economic fluctuations (http://www.economicsdiscussion). Samuelson believes that multiplier and accelerator interactions lead to cyclical fluctuations in economic activity. An increase in autonomous investment leads to an amplified rise in income, which is determined by the value of the multiplier (k) is the basic tenet of the Samuelson model. The multiplier (k) in the closed economy is written as

$$k = \frac{Change\ in\ Income}{Change\ in\ Investment} = \frac{\Delta Y}{\Delta I} = \frac{1}{Marginal\ Propensity\ to\ Save\ (MPS)}$$

$$= \frac{1}{1 - Marginal\ Propensity\ to\ Consume\ (MPC)}$$

Through accelerator effect an increase in autonomous investment (Ia) increase income level which in turn increases induced investment. The capital-output ratio is also known as accelerator (a) which is reverse of the multiplier. An increase in income level raises aggregate demand. The economy requires extra investment in capital goods to meet the increased aggregate demand. This induced investment is the result of extra investment taken by the government, and thus, induced income and autonomous investment mutually interact each other. This interaction between autonomous investment and induced investment affects the process of income and employment in a cyclical manner. This is what Samuelson model shows – combination of accelerator with the Keynesian multiplier results in an increase in income and output/ employment by even greater amount. This is shown in the Figure 1.

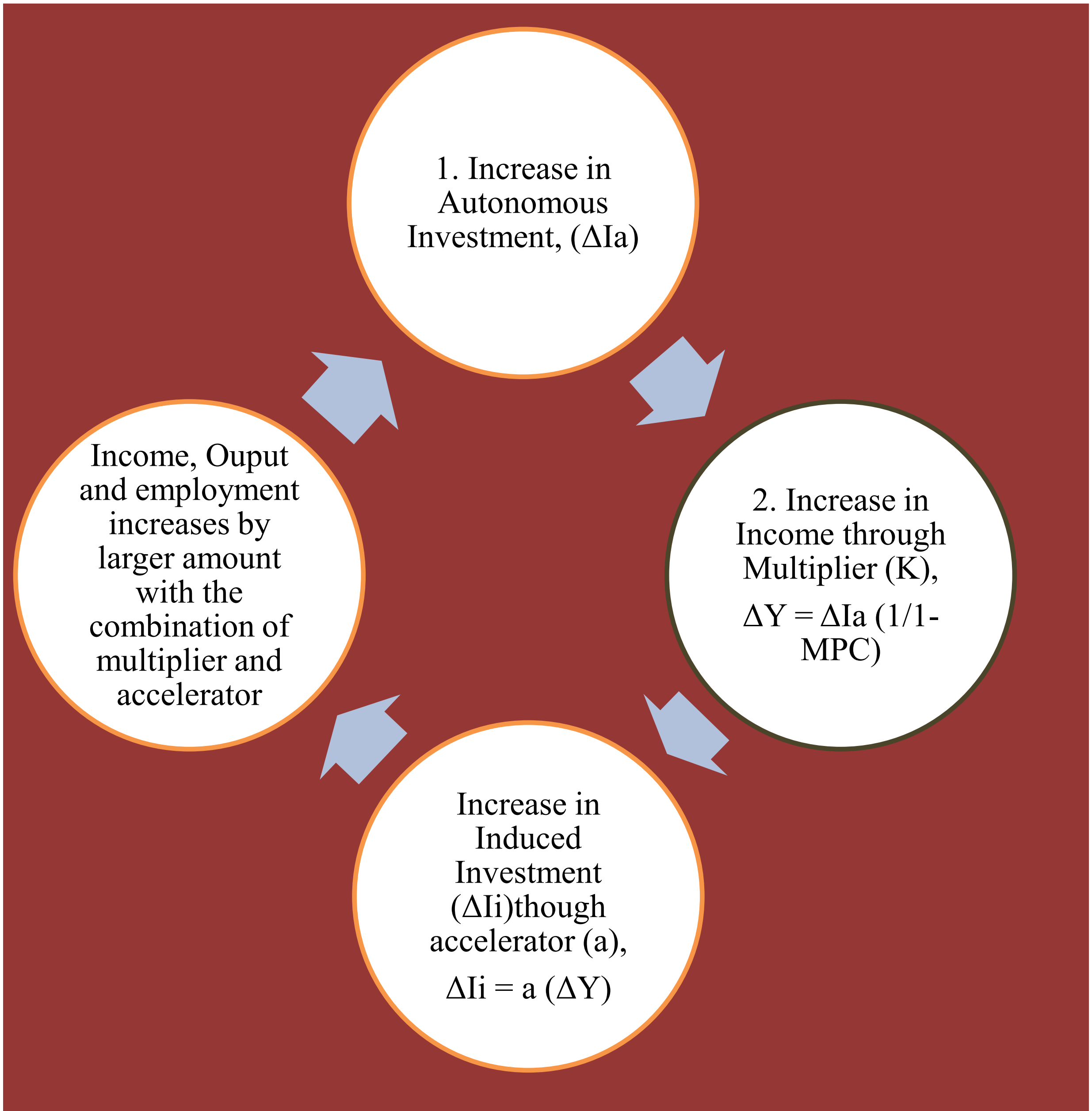

Figure 1: Multiplier and Accelerator Interaction

The accelerator and multiplier interaction model is mathematically represented as under:

$Y_t = C_t + I_t$ i
$C_t = C_a + c\,(Y_{t-1})$ ii
$I_t = I_a + a\,(Y_{t-1} - Y_{t-2})$ iii
$Y_t = C_a + c\,(Y_{t-1}) + I_a + a\,(Y_{t-1} - Y_{t-2})$ iv

Where, income, consumption and investment for a current period t, are shown by Y_t, C_t, and I_t respectively. Autonomous consumption is shown by C_a and autonomous investment is shown by

I_a. Capital-output ratio or accelerator is represented by a while c represents marginal propensity to consume. It must be noted here that government activity is not taken because Samuelson model neither considers the effects of public spending on income level and output level nor determinants of the public spending.

From above equation it is evident that:

1. **Equation i** shows that income in current period (Y_t) is the summation of Consumption in a current period (C_t) and investment in current period (I_t).
2. **Equation ii** shows that consumption in current period (C_t) is the summation of autonomous consumption (C_a) and function of previous year's income multiplied by marginal propensity to consume {c (Y_{t-1})}. This equation can also be written as $C_t = c\,(Y_{t-1})$. This shows that consumption in current period is a function of previous year's income. Equation ii shows that consumption is a linear function of the previous year, where marginal propensity to consume (c) is greater than zero but less than one, i.e., $0 < c < 1$.
3. **Equation iii** shows that induced investment in current period (I_t) is a summation of autonomous investment (I_a) and function of a change in income of the two previous years (i.e., $Y_{t-1} – Y_{t-2}$) multiplied by accelerator or capital- Output ratio, shown by a. Thus, induced investment is a product of change in income and accelerator, which is shown by {$a(Y_{t-1}– Y_{t-2})$}. Hence, $I_t = a(\Delta Y_{t-1})$ can also represent equation (iii). This shows that in current period induced investment is a function of change in income in the previous period. Put it differently, induced investment in current period depends on changes in income from two previous period. Equation iii indicates that investment is proportional to the change in the level of income/consumption between the previous and current period. In this equation iii a is accelerator or capital-output ratio, which is shown as,

$$Accelerator\ (a) = \frac{Investment}{Income/Consumption}.$$

Accelerator is assumed to be greater than zero, i.e., $a > 0$. It is assumed to be positive. Equation iii can also be written as,

$$I_t = I_a + a\,(C_t – C_{t-1}) \qquad \text{iii}$$

$$I_t = I_a + a\,(Y_{t-1} – Y_{t-2}) \qquad \text{since, } C_{t-1} = c\,(Y_{t-2}). \qquad \text{iii}$$

By replacing equations (ii) and (iii) in equation (i), we obtain the following equation iv,

$$Y_t = C_a + c\,(Y_{t-1}) + I_a + a\,(Y_{t-1} – Y_{t-2}) \qquad \text{iv}$$

Aggregate output or income in the current period (Yt) is determined by the sum of autonomous consumption (Ca), consumption based on the marginal propensity to consume (c) times previous year's income, autonomous investment (Ia), and the induced investment, which is given by the accelerator (a) multiplied by the change in income between the previous year and the year before that (Yt-1 – Yt-2), as equation (iv) shows. Hence, change in the level of income is determined by both capital-output ratio or accelerator (a) and the value of marginal propensity to consume (c), as equation (iv) indicates. Samuelson model states that if we know the value of marginal propensity to consume (c), accelerator (a), income of the current period (Y_t) and income of the previous period (Y_{t-1}); then the present and future income can be determined by putting the given values in the equation. It must be known here that, the level of income determined in the above equations show dynamic equilibrium. The determined level of income in static equilibrium will be

$$Y = C_a + c(Y) + I \qquad \text{v}$$

In static equilibrium, $Y_t = Y_{t-1} = Y_{t-2} = Y_{t-n}$. This shows that in static equilibrium period lags have no influence and thus equilibrium level of income remains unchanged. Consequently, accelerator reduced to zero. In contrast, in a dynamic state change in autonomous investment takes economy either towards a final equilibrium or diverging from the equilibrium level, as equation (iv) describes this path. The capital-output ratio or accelerator (a) and the values of marginal propensity to consume (c) determine if the economy will remain at new equilibrium point or moves away from it.

Samuelson has described five different paths of business cycle with the different combination of given value of marginal propensity to consume (c) and capital- output ratio {(or accelerator (a)} as shown in Table 1. The magnitude of multiplier and accelerator is determined by various combinations of the values of marginal propensity to consume and capital-output ratio, which are shown in Table 1.

Table 1: Samuelson's Five Different Path of Cycle

Region	Marginal Propensity to Consume (c)	Multiplier (k) $k = \frac{1}{1-c}$	Accelerator (a)	Different Paths of Trade/Business cycle	Types of Oscillation
A	0.5	2	0	Smooth Convergence	No Oscillation/Stable
B	0.5	2	1	Damped Cycle	Stable Oscillation

E	0.5	2	2	Constant Cycle (special case)	Constant Oscillation
C	0.5	2	3	Anti-Damped Cycle/Explosive Cycle	Unstable Oscillation
D	0.5	2	4	Smooth Expansion (Explosive Growth)	Unstable No Oscillation

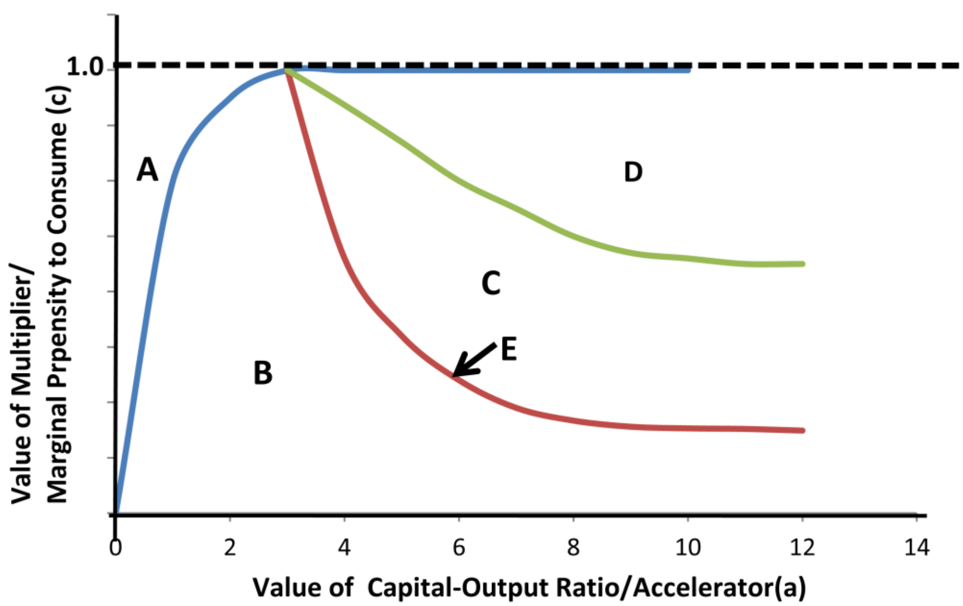

Figure 2: Samuelson Model of Business Cycle: Dynamic Income Path

Samuelson in his paper explained that movements in the economic activities depend on the values of marginal propensity to consume (c) and accelerator (a). With these given values, and given change in autonomous spending, he explained four cases of different dynamic income paths or five cases of dynamic income path if we include a special case, which is shown in Figure 2. Figure 2 shows five region of movement of income, viz., Region A, B, C, D, and E. Region A shows that when value of multiplier is 2 and accelerator is zero, with a change in autonomous investment or consumption, income or gross national product may move upward or downward with decreasing rate and reaches its new equilibrium point and become parallel to new equilibrium point as depicted in panel A of Figure 3. There is no oscillation in region A for new equilibrium is parallel to the initial equilibrium, and thus there in no cycle as shown in panel A of Figure 3.

With the change in autonomous consumption or investment, when values of accelerator increase to 1 and values of multiplier remain as before it generates fluctuation in the income and pattern of development in the income forms the shape of Region B as it is depicted in the Figure 2. With change in autonomous consumption or investment, values of multiplier and accelerator in region B cause income to fluctuate in the series of a damped cycles whose amplitudes gradually shrink until the cycles vanish, as depicted in panel B of Figure 3. In panel B of Figure 3, fluctuation in income generates stable oscillation as it disappeared on reaching the new equilibrium point.

In the region C of Figure 2 value of accelerator is greater than multiplier, consequently combination of marginal propensity to consume/ multiplier and accelerator or capital-output ratio generates fluctuation in income with successively greater and greater amplitude. This type of fluctuation in income is explosive in nature and with the passage of time its oscillation becomes unstable as multiplier become larger and larger. This is called anti-damped cycle or explosive cycle depicted in panel C of Figure 3. This explosive cycle must be restrained by ceiling and floor in case of expansion and contraction of business cycle respectively.

The values of multiplier or marginal propensity to consume (c) and accelerator (a) in the region D of Figure 2 generate high fluctuation in the income with successively greater and greater amplitude that cause the economic system to explode and diverge from the equilibrium state by an increasing amount. The value of multiplier and accelerator in this region may cause income to move upward or downward at an increasing rate. This depicts smooth expansion in the income with greater amplitude, which must be restrained by the factors determining ceiling and floor if the cyclical movements are to occur in the system. This is shown in panel D of Figure 3.

There is a special case of business cycle, which is shown by point E which lies between the boundary area of B and C. To understand this, you just draw a parallel curve to the curve marked by E. The value of multiplier and accelerator lies in area E generates fluctuation in income of constant amplitude as depicted in panel E of Figure 3.

It is worth noting here that all combinations of values of marginal propensity to consume (c) and accelerator (a) do not produce business cycle or cyclical fluctuation in the economy. Those combinations of marginal propensity to consume and accelerator lying in regions B, C, and E produce business cycle. In the region A and D of Figure 2, there is no business cycle because the combination of multiplier and accelerator in the region A are such that with changes in autonomous investment or autonomous consumption, income moves smoothly from initial equilibrium to new equilibrium without producing any oscillations (see panel A of Figure 3) in the economy. Similarly, the values of multiplier and accelerator in the region D are such that with changes in autonomous investment or autonomous consumption, income moves explosively in upward direction without producing any oscillation in the economic system. There is smooth expansion in income of region D, thus there is no cyclical fluctuation in region D.

The value of multiplier and accelerator in region B with changes in autonomous investment/consumption produces damped cycle. The oscillation produces in the region B are called damped cycles because this fluctuation tends to disappear over time or die out over period of time. This indicates that amplitude of cycle shrinks to zero and formation of cycle dies out. Generally, business cycle observed in the post-war period was relatively damped cycles as compare to that of inter-war period. However, there is no historical evidence to support the view that the business cycles will completely die out or disappear over period of time (https://shishirshakya.blogspot.com). This may happen only when one time investment takes place in the economy. In other words, business cycle in the region B explains the impact of one-time autonomous investment. If no disturbance takes place in the economy, the impact of one-time autonomous investment goes on declining over period of time. However, advancement in technology, innovation, natural disaster, and man-made crisis like security scam in India during 1991-92 occur in the economy quite frequently and at random intervals (http://www.economicsdiscussion.net). Thus, the values of multiplier and accelerator with such disturbances that takes place on and off in the economy tends to produce irregular business cycle which does not die out or disappear over period of time as mentioned in the region B of Figure 2.

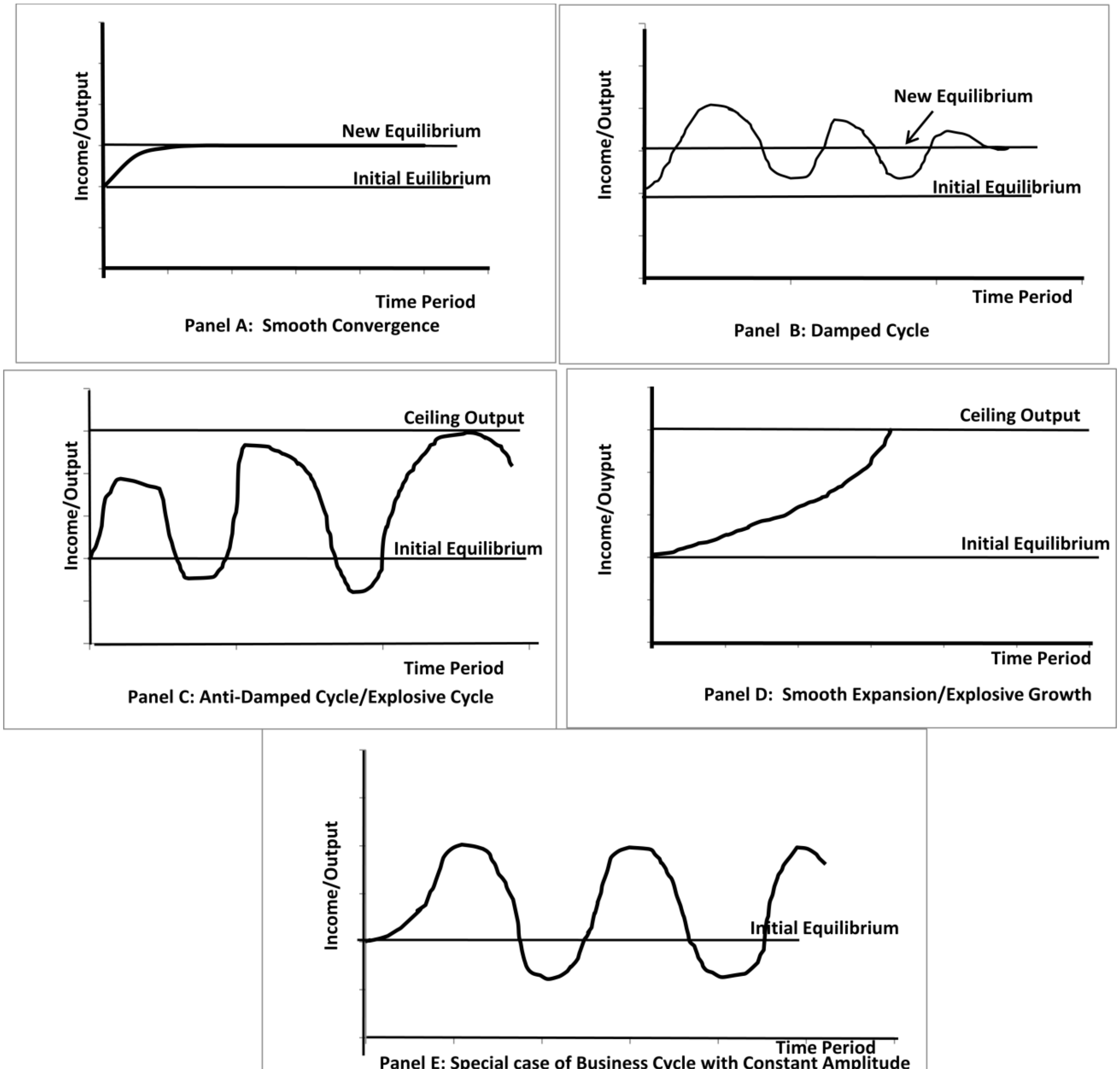

Figure 3: Different Pattersn of Income movement for various values of marginal propensity to consume and accelerator

The cycle generated by the value of multiplier and accelerator in the region C is of no doubt continued oscillation but this oscillation/cycle is explosive in nature, which is not in consistent with the real word situation where cycle does not become explosive. The government can make region C's explosive cycle consistent with the real world situation. By imposing ceiling or upper limit and floor or lower limit on the expansion and contraction of income and output respectively government may make explosive cycle in consistent with the real world situation

The region E produces such an oscillation or cycle which neither disappears nor explodes over period of time but goes on with constant amplitude, which does not fit in the real world.

Examination of Samuelson's Business Cycle

In Figure 2 and Figure 3, we have seen Samuelson's five cases of business cycle with the values of marginal propensity to consume, accelerator and autonomous investment. In this section, we observe some different business cycle after plotting the value of marginal propensity to consume, accelerator, and autonomous investment. The different income level is shown in Table 2. For calculation of income level see appendix. When we plot Figure 2 on the basis of Table 2, we get totally different figure, which is depicted in the Figure 4. Similarly, when we plot Figure 3, we get something different figure, which is depicted in Figure 5.

Table 2: Income/Output with the constant value of Marginal Propensity to Consume (c) and varying Capital-output Ratio or Accelerator (a)

Period	Income of Region A c = 0.5; a = 0	Income of Region B c = 0.5; a = 1	Income of Region E c = 0.5; a = 2	Income of Region C c = 0.5; a = 3	Income of Region D c = 0.5; a = 4
1	1	1	1	1	1
2	1.5	2	2.5	3.0	3.5
3	1.75	2.5	3.75	4.5	7.75
4	1.875	2.50	4.125	4.75	13.375
5	1.9375	2.25	3.4375	3.625	18.9375
6	1.9688	2.00	2.0314	1.6875	21.5938
7	1.9844	0.075	0.6095	-0.0939	17.1093
8	1.9922	0.975	-0.1171	-0.8282	0.5859
9	1.9961	1.5325	0.2147	-11.329	-31.754
10	1.9981	2.045	0.7755	-7.7115	-79.557

Note: Autonomous investment/ income of the period 1 is given as Rs.1.

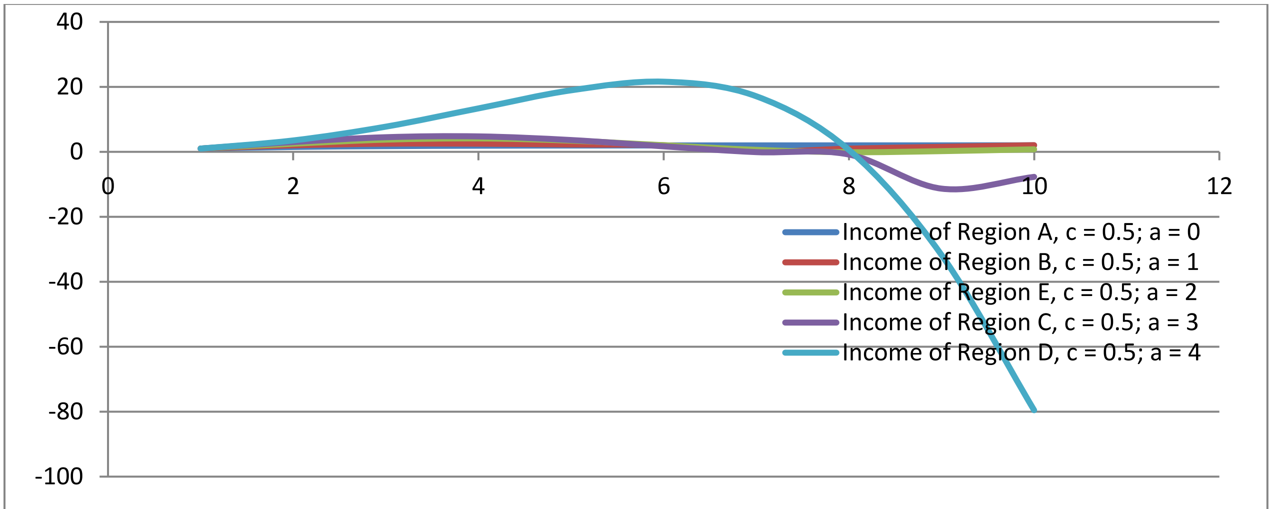

Figure 4: Business Cycle with the given value of marginal propensity to consume (c) and accelerator (a)

In Figure 5, business cycle shown in region A is similar to the business cycle of Samuelson shown in panel A of Figure 3. According to Samuelson region B of Figure 2 will form damped cycle that is the amplitude of fluctuation in income will shrink to zero as it is depicted in the panel B of Figure 3. However, damped cycle shown in region B is not damped as depicted by Samuelson as shown in panel B of Figure 3. Though, amplitude of fluctuation shrinks to zero but it starts moving from there in upward direction, thus, it cannot be named as damped cycle because the cycle does not disappear or die out after reaching to point zero as shown in region B of Figure 5.

Business cycle in Region E of Figure 5 matched with the business cycle of Samuelson as shown in panel E of Figure 3. The business cycle in both figures moves with constant amplitude. According to Samuelson business cycle in region C as shown in Figure 2 is explosive in nature. The oscillation in the region C is unstable and explosive and, thus, it requires ceiling in upward movement of business cycle as shown in panel C of Figure 3. However, when we plot the value of marginal propensity to consume and accelerator on the graph though we get explosive business cycle but it does not require ceiling as recommended by Samuelson because explosive business cycle declines over period of time as depicted in region C of Figure 5.

Region D shows smooth expansion in the level of income without any oscillation. Like region C, business cycle in region D is also explosive in nature and requires ceiling by the government. However, the cycle shown in region D of Figure 5 does not support the business cycle shown in panel D of Figure 3 because business cycle in region D of Figure 5 reflects smooth expansion but does not require ceiling as suggested by Samuelson.

The business cycle of region A and region E of Figure 5 fully support business cycle theory of the Samuelson. However, region B, C, and D of Figure 5 partially matched with the business cycle of Samuelson.

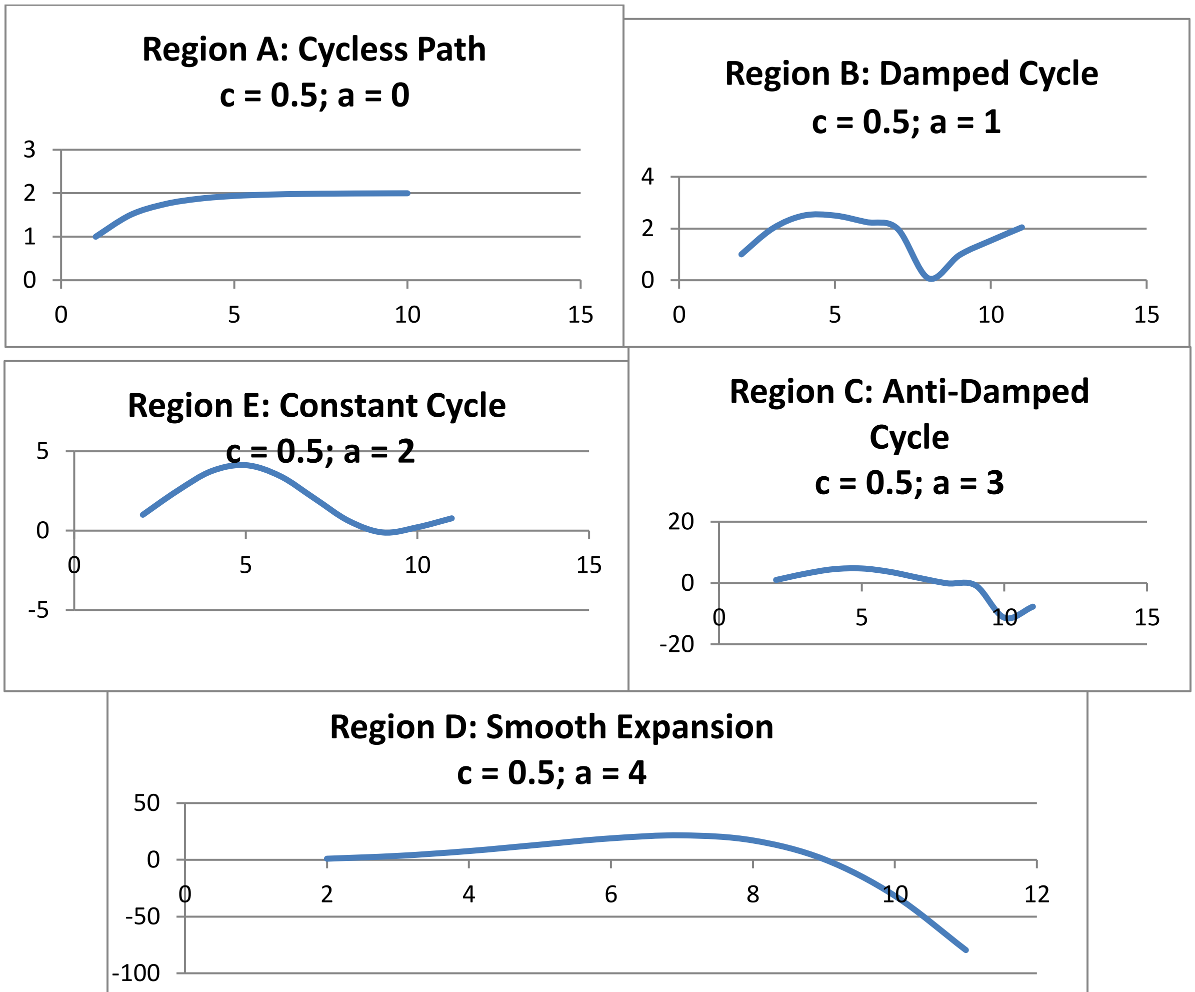

Figure 5:Pattern of Income Level with the given value of c and a

Table 3: Income/Output with the varying value of Marginal Propensity to Consume (c) and Capital-output Ratio or Accelerator (a)

Period	Income c = 0.5; a = 0	Income c = 0.5; a = 02	Income c = 0.6; a = 02	Income c = 0.8; a = 04
1	1	1	1	1
2	1.5	2.5	2.8	5
3	1.75	3.75	4.84	17.8

4	1.875	4.125	6.352	56.2
5	1.9375	3.4375	6.6256	169.84
6	1.9688	2.0313	5.3037	500.52
7	1.9844	0.9141	2.5959	1459.592
8	1.9922	0.1172	-0.6918	4227.704
9	1.9961	0.2148	-3.3603	12241.12

Source:http://www.economicsdiscussion.net/business-cycles/samuelsonsmodel/samuelsons-model-of-business-cycle-with-diagrams/10437.

Based on varying value of marginal propensity to consume and accelerator given in Table 3, we will get the business cycles as shown in Figure 6, which are more in consistent with the Samuelson model of business cycle explained in the Figure 3.

When marginal propensity to consume is 0.5 and accelerator is zero, income level will approach the peak. Since coefficient of accelerator is zero, this will be the case of multiplier effect only where income reaches asymptotic level as depicted in Figure 6.

When marginal propensity to consume is 0.5 and accelerator is 02, a regular cycle or continuous cycle with constant amplitude, i.e., the value of multiplier level is more or less unchanged repeating themselves indefinitely is arrived as depicted in Figure 6.

An explosive cycle is arrived with the value of marginal propensity to consume 0.6 and accelerator 02 as depicted in Figure 6. In this case variations in multiplier level becoming large and large over period of time, which result in explosive cycle.

The combination of marginal propensity to consume 0.8 and accelerator 04 will result in smooth expansion in income, where income gradually approaching a compound interest rate of growth as depicted in Figure 6.

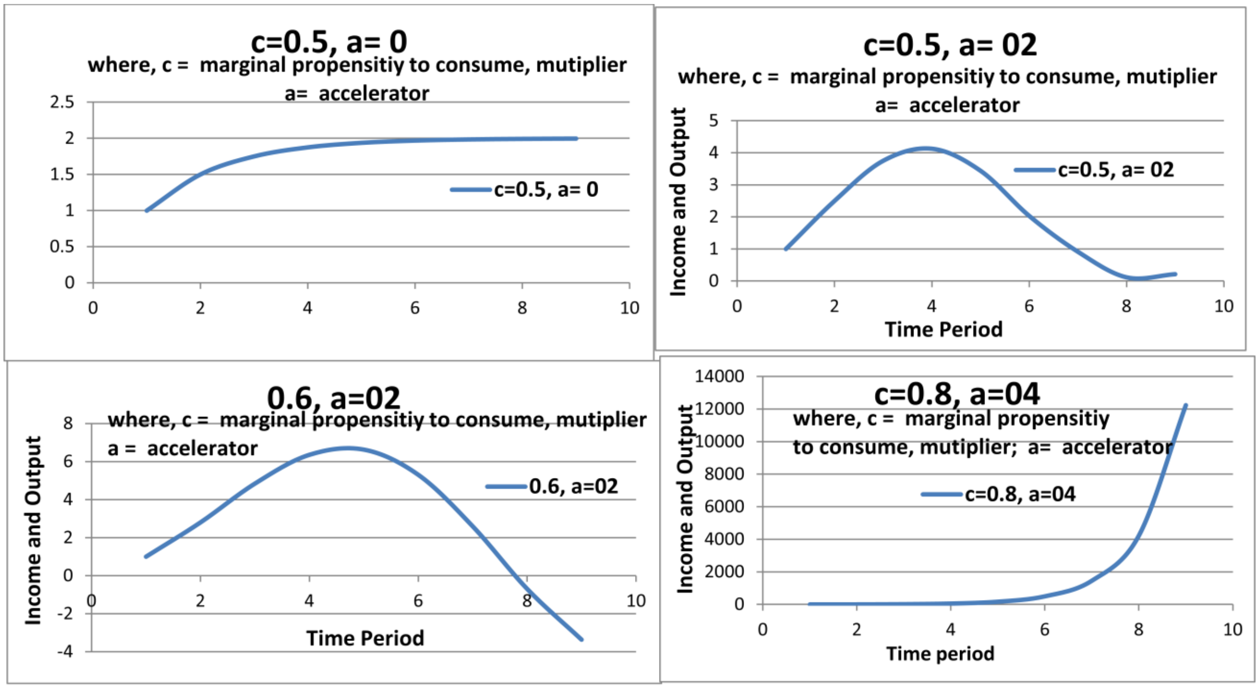

Figure 6: Different Pattern of Income Level with varying value of c and a

Multiplier-Accelerator Interaction

Paul Samuelson theory of business cycle is also known as multiplier-accelerator model of business cycle. Though many economists have tried to explain economic phenomenon including business cycle through multiplier-accelerator principles, Samuelson was pioneer among them (http://www.economicsdiscussion.net). Samuelson business cycle through multiplier-accelerator principles can be understood with the help of Table 4. In the formulation of Table 4, it is assumed that marginal propensity to consume (c) is equal to 0.5, and capital-output ratio or accelerator (a) is equal to 2. As we know that Samuelson model of business cycle is based on the assumption of one period time-lag, which states that an increase in consumption in next time period (t+1) is the result of an increase in income in one period (t). Initially it is assumed that in period t autonomous investment is Rs. 100. This autonomous investment will remain constant at Rs. 100 crores throughout the period. With this autonomous investment of Rs. 100 cores in period t, consumption in period t+1 will be increased by the value of marginal propensity to consume (c) which is 0.5. Thus, induced consumption in period t+1 = c (Y_{t-1}) = 0.5 (100) = Rs.50 as shown in column 3. Induced investment is the multiplication of accelerator with the previous consumption. Thus, induced investment in period t+1 = a (Ct - Ct-1) = 2 (50) = Rs. 100 as shown in column 4. Total income equals Rs. 250, which is the summation of autonomous investment, induced consumption, and induced investment. Similarly, in period t+2, induced consumption = c (Y_{t-1}) = 0.5 (250) = 125. Induced investment in period t+2 = a (Ct - Ct-1) = 2 (75) = Rs.150. Previous year's consumption of t+2 is the difference between t+1 and t, i.e., 125 – 50 = 75. Similarly, total income for rest of the period will be calculated on the basis of equation. When total income of Table 4 is plotted on graph, business cycle will look like as shown in Figure 7.

Table 4: Multiplier-Accelerator Interaction Model
Rs. in Crore

Period (t) (1)	Initial/Autonomous Investment (2)	Induced Consumption $C = c(Y_{t-1})$ c = 0.5 (3)	Induced Investment I = a (Ct - Ct-1) a = 2 (4)	Total Income (Y) (5)
t	100	0	0	100
t+1	100	50	100	250
t+2	100	125	150	375
t+3	100	187.5	125	412.5
t+4	100	206.2	37.4	343.6
t+5	100	171.8	-68.8	203
t+6	100	101.5	-140.6	60.9
t+7	100	30.4	-142.2	-11.8
t+8	100	-5.9	-72.6	21.9
t+9	100	10.7	33.2	143.9

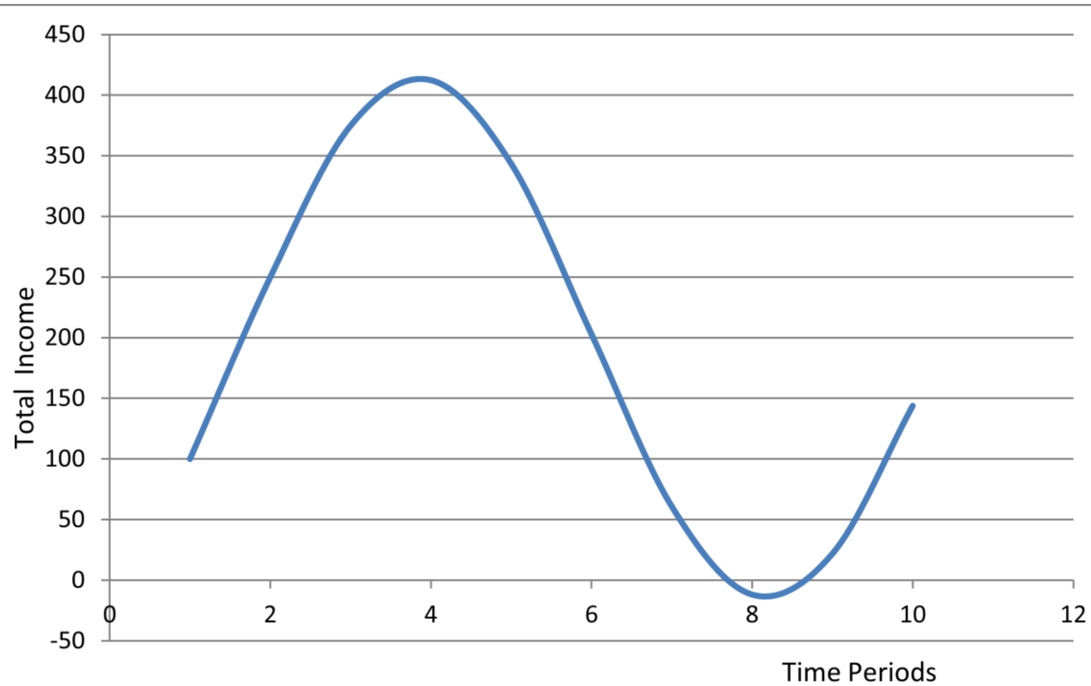

Figure 7: Business Cycle: Interaction of Multiplier and Accelerator

Limitation of Samuelson's Business Cycle

Samuelson model of business cycle is based on the assumption that interaction of multiplier and accelerator generates fluctuations in the pattern of income level and on that basis, he defined five cases of business cycle. With oscillation in region C and without oscillation in the region D (i.e., there is no business cycle in the region D because of smooth expansion in the pattern of income level), the values of multiplier and accelerator in the region C and D are such that they give rise to directly explosive upward or downward movement. The movements in the pattern of income level are restrained by ceiling in case of upward movements and by floor in case of downward movements (https://www.google.com/search). After reaching ceiling point, movements in the pattern of income level starts moving in the reverse direction. Samuelson's business cycle theory fails to explain this reverse direction. Business cycle theory of Hicks explained the causes for the movements in the pattern of income level in the reverse direction after reaching peak point/ceiling point.

The interaction of multiplier and accelerator principles no doubt plays an important role in the explanation of cyclical fluctuation in the pattern of income level. However, it does not go to raise the national income to higher and higher level because there are many practical difficulties in the calculation of total effects of interaction of multiplier and accelerator principles on the pattern of income level.

This model considers exogenous spending, i.e., government spending as constant and no attempt has been made to use government spending as an instrument for controlling the time path of income. In other words, this model ignores both the impact of changes in public spending on the level of income and determinants of the public spending (http://www.economicsdiscussion.net). Thus, this model is based on the structure of time-lag and coefficients of multiplier and accelerator which relate the variables in the equations.

Conclusion

We arrive at the process of income generation through the working together of multiplier and accelerator principles. However, Paul Samuelson went further and stated that through the values of multiplier and accelerator, it is possible to explain the path of fluctuations in the pattern of income level and on that basis, he defined five cases of business cycle. Paul Samuelson stated that if we know autonomous consumption, autonomous investment, consumption for the current period and preceding period, we can determine the growth path of income and output for any period with the given values of marginal propensity to consume and capital-output ratio or accelerator by substituting these values in the equation of income (Y).

Appendix

Table 2: Explanation of Table 1

Income of Region A **When c = 0.5 and a = 0**	**Income of Region B** **When c = 0.5 and a = 1**
Period 2 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ $Y_t = 0.5$ (1) + 0 = 0.5 Y_t = 1 + 0.5 = 1.5	Period 2 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ $Y_t = 0.5$ (1) + 1(0.5) = 1.0 Y_t = 1 + 1.0 = 2.0
Period 3 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ Y_t = 0.5 (1.5) + 0 = 0.75 Y_t = 1 + 0.75 = 1.75	Period 3 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ Y_t = 0.5 (2) + 1(0.5) Y_t = 1.0 + 0.5 = 1.5 Y_t = 1 + 1.5 = 2.5 Note: $C_t – C_{t-1}$ = 1.0 - 0.5 = 0.5
Period 4 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ Y_t = 0.5 (1.75) + 0 = 0.875 Y_t = 1 + 0.875 = 1.875	Period 4 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ Y_t = 0.5 (2.5) + 1(0.25) Y_t = 1.25 + 0.25 = 1.50 Y_t = 1 + 1.50 = 2.50 Note: $C_t – C_{t-1}$ = 1.25 – 1.0 = 0.25
Period 5 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ Y_t = 0.5 (1.875) + 0 = 0.9375 Y_t = 1 + 0.9375 = 1.9375	Period 5 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ Y_t = 0.5 (2.5) + 1(0) Y_t = 1.25 + 0 = 1.25 Y_t = 1 + 1.25 = 2.25 Note: $C_t – C_{t-1}$ = 1.25 – 1.25 = 0
Period 6 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ Y_t = 0.5 (1.9375) + 0 = 0.96875 Y_t = 1 + 0.96875 = 1.96875	Period 6 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ Y_t = 0.5 (2.25) + 1(- 0.125) Y_t = 1.125 – 0.125 = 1.00 Y_t = 1 + 1.00 = 2.00 Note: $C_t – C_{t-1}$ = 1.125 – 1.25 = - 0.125
Period 7 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ Y_t = 0.5 (1. 96875) + 0 = 0.984375 Y_t = 1 + 0.984375 = 1.984375 = 1.9844	Period 7 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ Y_t = 0.5 (2.00) + 1(-1.025) Y_t = 0.1 – 1.025 = -0.925 Y_t = 1 + (-0.925) = 0.075 Note: $C_t – C_{t-1}$ = 0.1 – 1.125 = -1.025
Period 8 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ Y_t = 0.5 (1.9844) + 0 = 0.9922 Y_t = 1 + 0.9922 = 1.9922	Period 8 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ Y_t = 0.5 (0.075) + 1(-0.0625) Y_t = 0.0375 – 0.0625 = -0.025 Y_t = 1 + (-0.025) = 0.975 Note: $C_t – C_{t-1}$ = 0.0375 – 0.1= -0.0625
Period 9 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ Y_t = 0.5 (1.9922) + 0 = 0.9961 Y_t = 1 + 0.9961 = 1.9961	Period 9 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ Y_t = 0.5 (0.975) + 1(0.45) Y_t = 0.4875 + 0.045 = 0.5325 Y_t = 1 + 0.5325 = 1.5325 Note: $C_t – C_{t-1}$ = 0.4875 – 0.0375 = 0.45
Period 10 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ Y_t = 0.5 (1.9961) + 0 = 0.9981 Y_t = 1 + 0.9981 = 1.9981	Period 10 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ Y_t = 0.5 (1.5325) + 1(0.27875) Y_t = 0.76625 + 0.27875= 1.045 Y_t = 1 + 1.045 = 2.045 Note: $C_t – C_{t-1}$ = 0.76625 – 0.4875 = 0.27875
Income of Region E **When c = 0.5 and a = 2**	**Income of Region C** **When c = 0.5 and a = 3**

Period 2 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$ $Y_t = 0.5\ (1) + 2(0.5)$ $Y_t = 0.5 + 1.0 = 1.5$ $Y_t = 1 + 1.5 = 2.5$	Period 2 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$ $Y_t = 0.5\ (1) + 3(0.5)$ $Y_t = 0.5 + 1.5 = 2.0$ $Y_t = 1 + 2.0 = 3.0$
Period 3 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$ $Y_t = 0.5\ (2.5) + 2(0.75)$ $Y_t = 1.25 + 1.50 = 2.75$ $Y_t = 1 + 2.75 = 3.75$ Note: $C_t - C_{t-1} = 1.25 - 0.5 = 0.75$	Period 3 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$ $Y_t = 0.5\ (3) + 2(1.0)$ $Y_t = 1.5 + 2.0 = 3.5$ $Y_t = 1 + 3.5 = 4.5$ Note: $C_t - C_{t-1} = 1.5 - 0.5 = 1.0$
Period 4 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$ $Y_t = 0.5\ (3.75) + 2(0.625)$ $Y_t = 1.875 + 1.250 = 3.125$ $Y_t = 1 + 3.125 = 4.125$ Note: $C_t - C_{t-1} = 1.875 - 1.25 = 0.625$	Period 4 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$ $Y_t = 0.5\ (4.5) + 2(0.75)$ $Y_t = 2.25 + 1.50 = 3.75$ $Y_t = 1 + 3.75 = 4.75$ Note: $C_t - C_{t-1} = 2.25 - 1.5 = 0.75$
Period 5 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$ $Y_t = 0.5\ (4.125) + 2(0.1875)$ $Y_t = 2.0625 + 0.375 = 2.4375$ $Y_t = 1 + 2.4375 = 3.4375$ Note: $C_t - C_{t-1} = 2.0625 - 1.875 = 0.1875$	Period 5 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$ $Y_t = 0.5\ (4.75) + 2(0.125)$ $Y_t = 2.375 + 0.250 = 2.625$ $Y_t = 1 + 2.625 = 3.625$ Note: $C_t - C_{t-1} = 2.375 - 2.25 = 0.125$
Period 6 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$ $Y_t = 0.5\ (3.4375) + 2(-0.3437)$ $Y_t = 1.7188 - 0.6874 = 1.0314$ $Y_t = 1 + 1.0314 = 2.0314$ Note: $C_t - C_{t-1} = 1.7188 - 2.0625 = -0.3437$	Period 6 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$ $Y_t = 0.5\ (3.625) + 2(-0.5625)$ $Y_t = 1.8125 - 1.125 = 0.6875$ $Y_t = 1 + 0.6875 = 1.6875$ Note: $C_t - C_{t-1} = 1.8125 - 2.375 =$ -0.5625
Period 7 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$ $Y_t = 0.5\ (2.0314) + 2(-0.7031)$ $Y_t = 1.0157 - 1.4062 = -0.3905$ $Y_t = 1 + (-0.3905) = 0.6095$ Note: $C_t - C_{t-1} = 1.0157 - 1.7188 = -0.7031$	Period 7 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$ $Y_t = 0.5\ (1.6875) + 2(-0.9688)$ $Y_t = 0.8436 - 1.9375 = -1.0939$ $Y_t = 1 + (-1.0939) = -0.0939$ Note: $C_t - C_{t-1} = 0.8436 - 1.8125 =$ -0.9688
Period 8 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$ $Y_t = 0.5\ (0.6095) + 2(-0.7110)$ $Y_t = 0.3048 - 1.4219 = -1.1171$ $Y_t = 1 + (-1.1171) = -0.1171$ Note: $C_t - C_{t-1} = 0.3048 - 1.0157 = -0.7110$	Period 8 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$ $Y_t = 0.5\ (-0.0939) + 2(-0.8906)$ $Y_t = -0.0470 - 1.7812 = -1.8282$ $Y_t = 1 + (-1.8282) = -0.8282$ Note: $C_t - C_{t-1} = -0.0470 - 0.8436 =$ -0.8906
Period 9 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$ $Y_t = 0.5\ (-0.1171) + 2(-0.3634)$ $Y_t = -0.0586 - 0.7267 = -0.7853$ $Y_t = 1 + (-0.7853) = 0.2147$ Note: $C_t - C_{t-1} = -0.0586 - 0.3048 = -0.3634$	Period 9 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$ $Y_t = 0.5\ (-0.8282) + 2(-4.094)$ $Y_t = -4.141 - 8.188 = -12.329$ $Y_t = 1 + (-12.329) = -11.329$ Note: $C_t - C_{t-1} = -4.141 - (-0.0470) =$ -4.094
Period 10 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$ $Y_t = 0.5\ (0.2147) + 2(0.1660)$ $Y_t = 0.1074 - 0.3319 = -0.2245$ $Y_t = 1 + (-0.2245) = 0.7755$ Note: $C_t - C_{t-1} = 0.1074 - (-0.0586) = 0.1660$	Period 10 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$ $Y_t = 0.5\ (-11.329) + 2(-1.5235)$ $Y_t = -5.6645 - 3.047 = -8.7115$ $Y_t = 1 + (-8.7115) = -7.7115$ Note: $C_t - C_{t-1} = -5.6645 - (-4.141) =$ -1.5235
Region D, when c = 0.5 and, a = 4	
Period 2 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$ $Y_t = 0.5\ (1) + 4(0.5)$ $Y_t = 0.5 + 2.0 = 2.5$ $Y_t = 1 + 2.5 = 3.5$	Period 3 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$ $Y_t = 0.5\ (3.5) + 4(1.25)$ $Y_t = 1.75 + 5.00 = 6.75$ $Y_t = 1 + 6.75 = 7.75$ Note: $C_t - C_{t-1} = 1.75 - 0.5 = 1.25$
Period 4 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$	Period 5 $Y_t = c(Y_{t-1}) + a(C_t - C_{t-1})$

$Y_t = 0.5\ (7.75) + 4(2.125)$ $Y_t = 3.875 + 8.500 = 12.375$ $Y_t = 1 + 12.375 = 13.375$ Note: $C_t – C_{t-1} = 3.875 – 1.75 = 2.125$	$Y_t = 0.5\ (13.375) + 4(2.8125)$ $Y_t = 6.6875 + 11.25 = 17.9375$ $Y_t = 1 + 17.9375 = 18.9375$ Note: $C_t – C_{t-1} = 6.6875 – 3.875 = 2.8125$
Period 6 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ $Y_t = 0.5\ (18.9375) + 4(2.7813)$ $Y_t = 9.4688 + 11.125 = 20.5938$ $Y_t = 1 + 20.5938 = 21.5938$ Note: $C_t – C_{t-1} = 9.4688 – 6.6875 = 2.7813$	Period 7 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ $Y_t = 0.5\ (21.5938) + 4(1.3281)$ $Y_t = 10.7969 + 5,3124 = 16.1093$ $Y_t = 1 + 16.1093 = 17.1093$ Note: $C_t – C_{t-1} = 10.7969 – 9.4688 = 1.3281$
Period 8 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ $Y_t = 0.5\ (17.1093) + 4(-2.2422)$ $Y_t = 8.5547 + (-8.9688) = -0.4141$ $Y_t = 1 + (-0.4141) = 0.5859$ Note: $C_t – C_{t-1} = 8.5547 – 10.7969 = -2.2422$	Period 9 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ $Y_t = 0.5\ (0.5859) + 4(-8.2618)$ $Y_t = 0.2930 – 33.047 = -32.754$ $Y_t = 1 + (-32.754) = -31.754$ Note: $C_t – C_{t-1} = 0.2930 – 8.5547 =$ -8.2618
Period 10 $Y_t = c(Y_{t-1}) + a(C_t – C_{t-1})$ $Y_t = 0.5\ (-31.754) + 4(-16.17)$ $Y_t = -15.877 – 64.68 = -80.557$ $Y_t = 1 + (-80.557) = -79.557$ Note: $C_t – C_{t-1} = -15.877 – 0.2930 = -16.17$	

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